Integral Equations and State Variables in the S-Domain Modeling of Passive Multilayered Components

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Abstract— **This paper outlines the basic ideas of a new and very efficient integral method for the electromagnetic modelling of boxed multilayered passive circuits. Differently from standard integral approaches, the method leads to a state-space representation of the component, which directly permits us to find its admittance matrix, in the form of a reduced-order pole expansion in the s-domain, through standard Krylov sub-space technique.**

I. Introduction

A particular attention is presently devoted to the development of efficient electromagnetic solvers that directly lead to the mathematical model of passive components or subsystems, through the application of the so-called S-domain methods [1]. The mathematical model is obtained in the form of pole expansion of some circuit matrix (scattering, impedance, admittance, ...) in the domain of the Laplace variable s. Such models are very useful for representing passive components or subsystems in the design of complex integrated systems, carried out in a network-oriented simulation environment.

The basic problem encountered in S-methods derives from the necessity of representing a distributed structure, inherently of infinite order, by a macromodel of finite and reasonably small order. Among the many possible techniques suitable for this purpose, the most robust and effective are based on the Krylov sub-space methods, such as the matrix-Padè-via-Lanczos (MPVL) and the block Arnoldi algorithms, largely used in the reduced-order modelling of complex VLSI circuitry [2] [3].

Krylov sub-space methods apply to state-space models of linear systems and, for this reason, they are well-matched to the Finite Element Method, which gives rise to a model of this type [1] [4] [5]. On the contrary, Integral Equation Methods, which are the most effective in the analysis of planar components, do not give rise to equations in the state-space form, so that Krylov sub-space methods can be applied only through adaptive procedures [6], which partially reduce their efficiency.

In this paper we give an outline of a new S-domain integral method for the modelling of shielded multilayered passive MMIC components or subsystems. Differently from standard integral methods, the new procedure directly results into state-equations, thus permitting to exploit

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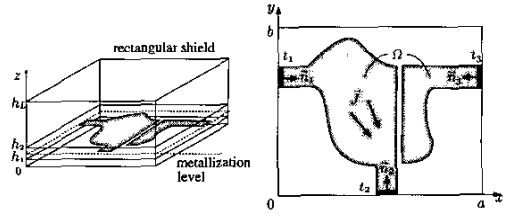

Fig. 1. Shielded multilayered microstrip circuit. The region Ω includes the metallization (shadowed area) and the delta-gaps (segments $t_1, t_2, \ldots$)

at best the advantages of the integral approach and of Krylov sub-space methods. Some preliminary conference papers were already published [9]-[11], which presented a similar method, valid only in the case of components on low-loss substrates. The algorithm presented here has been modified, in order to consider also lossy substrates, and to reformulate the problem in terms of real state-variables, thus allowing the use of standard model-order reduction techniques [2]. A detailed description of the new method will be given in a forthcoming journal paper.

II. Outline of the Theory

Let us consider a shielded passive component consisting of thin metal elements embedded in a layered medium, including both insulating and semiconducting layers (Fig. 1). For the sake of simplicity we assume that these elements are located outside the semiconducting layers, at a single "metallization level". Furthermore, as usual, we assume a "delta-gap voltage excitation" [7] [8]. The shadowed area Ω shown in Fig. 1 represents the metallization, including the gaps $t_1, t_2, \ldots, t_N$ where the exciting voltages $v_1, v_2, \ldots, v_N$ are applied. The excitation gives rise to a surface current $\vec{J}$ distributed in the metallization, and to a set of gap-currents $i_1, i_2, \ldots, i_N$. The positive direction of the gap-voltages and currents is defined by the normals $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N$. We have:

$$i_n = \int_{t_n} \vec{J} \cdot \vec{n}_n dt_n \qquad (1)$$

Over the surface Ω the tangential electric field must satisfy the boundary condition:

$$\vec{E}_T(x,y) = -\sum_{n=1}^{N} v_n \delta_n \vec{n}_n + Z\vec{J}(x,y) \quad \forall\, x,y \in \Omega \qquad (2)$$

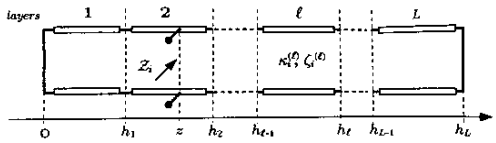

Fig. 2. Equivalent circuit for the calculation of $\mathcal{Z}_i$. The line sections correspond to the layers and z is the metallization level.

where δ_n denotes a delta-function supported by the segment t_n, and Z is the surface impedance of the metal elements. On the other hand the electric field is related to the currents density by the integral

$$\vec{E}_T(x,y) = -\int_0^a\int_0^b \overline{\overline{Z}}(x,y,x',y')\cdot\vec{J}(x',y')dx'dy' \quad (3)$$

where

$$\overline{\overline{Z}} = \sum_i \mathcal{Z}_i\,\vec{e}_i(x,y)\,\vec{e}_i(x',y') \quad (4)$$

where vectors $\vec{e}_i$ are the normalized electric mode-vectors of the modes of the rectangular waveguide of sides a, b and $\mathcal{Z}_i$ is the modal-impedance seen from the metallization level, looking in the layered rectangular cavity. For each mode, $\mathcal{Z}_i$ is calculated by considering the transverse equivalent circuit shown in Fig. 2, where each line-section corresponds to a layer and the terminal short circuits correspond to the conducting planes at $z = 0$ and $z = h_L$. The propagation factor ($\kappa_i^{(\ell)}$) and the characteristic impedance ($\zeta_i^{(\ell)}$) depend on the layer and on the mode. Their expressions in terms of the Laplace variable s are given in Table I, where σ_ℓ and ϵ_ℓ are the conductivity and the relative permittivity of the ℓ-th layer, and k_c is the cutoff wavenumber of the mode. On substitution of (3) into (2) we obtain an integral equation, whose solution yields the current density generated by a given set of voltages. Then, substituting into (1) we find the relationship between the voltages and the currents, i.e, the admittance matrix of the component.

The integral equation is solved by using the Method of the Moments. We approximate the current density in a finite dimensional functional space, using the expression

$$\vec{J} = \sum_{k=1}^{K} c_k\,\vec{w}_k(x,y) \quad (5)$$

where $\{\vec{w}_k\}$ is a set of real basis functions defined on Ω and $\{c_k\}$ is a set of complex variables. Using the Galerkin's method the integral equation is transformed into the matrix equation

$$\mathbf{Z}\,\mathbf{c} = \mathbf{T}\,\mathbf{v} \quad (6)$$

where $\mathbf{v}$ is the voltage vector, $\mathbf{c}$ is the vector of the variables c_k, and $\mathbf{Z} \in \mathcal{C}^{K\times K}$, $\mathbf{T} \in \Re^{K\times N}$ are matrices with entries given by

$$Z_{hk} = \sum_i w_{hi}w_{ki}\mathcal{Z}_i + ZK_{hk} \quad (7)$$

TABLE I

i-th mode	$\kappa_i^{(\ell)}$	$\zeta_i^{(\ell)}$
TM_{pq}	$\sqrt{-(s^2\epsilon_0\epsilon_\ell + s\sigma_\ell)\mu_o - k_c^2}$	$\dfrac{j\kappa_{pq}^{(\ell)}}{s\epsilon_0\epsilon_\ell + \sigma_\ell}$
TE_{pq}		$\dfrac{s\mu_o}{j\kappa_{pq}^{(\ell)}}$

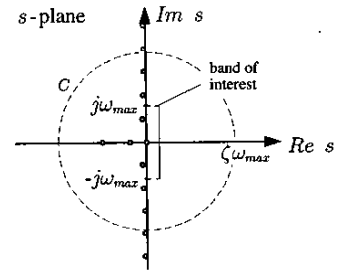

Fig. 3. Typical pole pattern of $\mathcal{Z}_i$ for a TM mode. The accuracy of the approximation (10) increases with increasing the value of the "accuracy factor" ζ. A value $\zeta = 2 \div 3$ is appropriate.

$$T_{hn} = \int_{t_n} \vec{w}_h\cdot\vec{n}_n\,dt_n \quad (8)$$

where $w_{hi} := \int_\Omega \vec{w}_h\cdot\vec{e}_i d\Omega$ and $K_{hk} := \int_\Omega \vec{w}_h\cdot\vec{w}_k d\Omega$. On the other hand, from (1) and (5) we obtain

$$\mathbf{i} = \mathbf{T}^T\,\mathbf{c} \quad (9)$$

To put (6) into the form of a state-equation we introduce the pole expansion of the modal impedance [12]. We have an infinity of complex pole-pairs (Fig. 3), corresponding to damped oscillating modes of the equivalent circuit of Fig. 2. Furthermore, we possibly have a finite number of real poles, corresponding to damped non-oscillating modes. With the low conductivities of the semiconductors normally used in MMICs, it is found that real poles are absent in TE-mode impedance, whereas they are still present in TM-mode impedance, in a number equal to the number of interfaces involving one or two semiconductor layers, i.e., a number of the order of unity.

Assuming that an accurate modeling is only required in some given "band of interest" $(0, \omega_{max})$, we can approximate someway the contribution of all poles far from the corresponding portion of the imaginary axis (see Fig. 3). Then, considering a circle C of radius $\zeta\omega_{max}$, sufficiently larger than ω_{max}, we can truncate the pole expansions by retaining the poles located inside C and approximating the contribution of all other poles by a power expansion around the origin, truncated to the first order. We have

$$\mathcal{Z}_i \approx \mathcal{R}_i + s\mathcal{L}_i + \frac{\mathcal{S}_i}{s} + \sum_C \frac{\beta_{i\nu}^2}{s+g_{i\nu}} + \sum_C\left(\frac{(\alpha'_{i\mu}+j\alpha''_{i\mu})^2}{s+r_{i\mu}-j\omega_{i\mu}} + \frac{(\alpha'_{i\mu}-j\alpha''_{i\mu})^2}{s+r_{i\mu}+j\omega_{i\mu}}\right) \quad (10)$$

where: $-g_{i\nu}$ and $(\beta_{i\nu})^2$ represent a real pole and its real residue; $-r_{i\mu} \pm j\omega_{i\mu}$ and $(\alpha'_{i\mu} \pm j\alpha''_{i\mu})^2$ represent a complex pole-pair and their residues; the indexes μ and ν label the poles in the order of increasing distances from the origin; Σ_C is a summation including only the poles located in C; $\mathcal{S}_i$ is the (real, non-negative) residue of the pole at the origin (not existing for TE modes); $\mathcal{R}_i$, $\mathcal{L}_i$ are real quantities and $\mathcal{R}_i + s\mathcal{L}_i$ approximates the contribution from all poles outside C. All coefficients included in the expansion depend only on the box and the metallization level, and can be determined once for all, and used for considering different metallization patterns. An efficient code for their calculation is described in [9].

The pole-set of the matrix $\mathbf{Z}$ consists of the complex and real poles ($\in C$) of all the impedances included in the summation (7). These poles are renamed by using a single-index notation, thus constructing the sequences $\{r_1, r_2, \ldots, r_{M'}\}$, $\{\omega_1, \omega_2, \ldots, \omega_{M'}\}$ and $\{g_1, g_2, \ldots g_{M''}\}$. The same indexing is used for the residues.

It is important to note that, with increasing the mode order i, the complex poles of the impedance $\mathcal{Z}_i$ move toward infinity and go outside the circle C. Then, the number M' of complex poles is finite (and reasonably small), whatever large is the number of terms included in the summation (7). The same statement does not hold true for real poles, because they move toward *finite* asymptotic values. If these values are placed inside of the circle C (which is likely to happen in the case of high resistivity semiconductor layers) the number M'' increases with increasing the number of terms included in the summation, and it can be very large. Equation (6) can be put into the form

$$[\mathbf{R} + Z\mathbf{K} + s\mathbf{L}]\mathbf{c} + \mathbf{S}\,\mathbf{d} + \mathbf{B}\mathbf{b} + \mathbf{A}'\mathbf{a}' + \mathbf{A}''\mathbf{a}'' = \mathbf{T}\mathbf{v} \quad (11)$$

where: $\mathbf{R}, \mathbf{K}, \mathbf{L}, \mathbf{S} \in \Re^{K\times K}$ are symmetric matrices of evident definition; matrices $\mathbf{A}'$, $\mathbf{A}'' \in \Re^{K\times M'}$ are defined as

$$A'_{km} = \sqrt{2}w_{kj}\alpha'_{j\mu} \qquad A''_{hm} = \sqrt{2}w_{hj}\alpha''_{j\mu} \qquad (m \mapsto j, \mu)$$

matrix $\mathbf{B} \in \Re^{K\times M''}$ is defined as

$$B_{km} = w_{kj}\beta_{j\nu} \qquad (m \mapsto j, \nu)$$

and the vectors $\mathbf{a}', \mathbf{a}'' \in \mathcal{C}^{M'}$, $\mathbf{b} \in \mathcal{C}^{M''}$, $\mathbf{d} \in \mathcal{C}^{K}$ satisfy

$$(\mathbf{D}' + s\mathbf{I}_{M'})\mathbf{a}' + \mathbf{D}\mathbf{a}'' - \mathbf{A}'^{\mathrm{T}}\mathbf{c} = 0 \quad (12)$$

$$(\mathbf{D}' + s\mathbf{I}_{M'})\mathbf{a}'' - \mathbf{D}\mathbf{a}' + \mathbf{A}''^{\mathrm{T}}\mathbf{c} = 0 \quad (13)$$

$$(\mathbf{D}'' + s\mathbf{I}_{M''})\mathbf{b} - \mathbf{B}^{\mathrm{T}}\mathbf{c} = 0 \quad (14)$$

$$s\mathbf{S}\,\mathbf{d} - \mathbf{S}\,\mathbf{c} = 0 \quad (15)$$

where $\mathbf{D} := \mathrm{diag}(\omega_1, \ldots, \omega_{M'})$, $\mathbf{D}' := \mathrm{diag}(r_1, \ldots, r_{M'})$, $\mathbf{D}'' := \mathrm{diag}(g_1, \ldots, g_{M''})$ and $\mathbf{I}_n$ denotes the identity matrix of order n. All matrices are independent of s.

For the sake of simplicity let us suppose that also Z is independent of s and assume that it is purely resistive (e.g., Z can be chosen as the surface resistance of the metallization at the frequency $\omega_{max}/2$). With this assumption, equations (11)-(15), explicitly exhibit the dependence on the parameter s, in the typical form of state-equations, where the state variables are represented by the elements of the vectors $\mathbf{a}', \mathbf{a}'', \mathbf{b}, \mathbf{c}, \mathbf{d}$. Introducing

$$\mathbf{M} := \begin{bmatrix} \mathbf{D}' & \mathbf{0} & \mathbf{0} & -\mathbf{A}'^{\mathrm{T}} & \mathbf{D} \\ \mathbf{0} & \mathbf{D}'' & \mathbf{0} & -\mathbf{B}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{S} & \mathbf{0} \\ -\mathbf{A}' & -\mathbf{B} & -\mathbf{S} & -\widehat{\mathbf{R}} & -\mathbf{A}'' \\ \mathbf{D} & \mathbf{0} & \mathbf{0} & -\mathbf{A}''^{\mathrm{T}} & -\mathbf{D}' \end{bmatrix}$$

$$\mathbf{N} := \begin{bmatrix} \mathbf{I}_{M'} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M''} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{S} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_{M'} \end{bmatrix} \quad \mathbf{x} := \begin{bmatrix} \mathbf{a}' \\ \mathbf{b} \\ \mathbf{d} \\ \mathbf{c} \\ \mathbf{a}'' \end{bmatrix} \quad \widehat{\mathbf{T}} := \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{T} \\ \mathbf{0} \end{bmatrix}$$

the mathematical model of the component is put into the standard state-space form

$$(\mathbf{M} + s\mathbf{N})\,\mathbf{x} = \widehat{\mathbf{T}}\,\mathbf{v} \quad (16)$$

$$\mathbf{i} = \widehat{\mathbf{T}}^{\mathrm{T}}\mathbf{x} \quad (17)$$

The order of the model, i.e., the number of state variables, is $M = 2K + 2M' + M''$. It can be large, especially due to the possible large values of M'' and K. Using a Krylov sub-space algorithm the original model is replaced by the reduced-order model

$$(\underline{\mathbf{M}} + s\underline{\mathbf{N}})\,\underline{\mathbf{x}} = \underline{\widehat{\mathbf{T}}}\,\mathbf{v} \quad (18)$$

$$\mathbf{i} = \underline{\widehat{\mathbf{T}}}^{\mathrm{T}}\,\underline{\mathbf{x}} \quad (19)$$

and the admittance matrix is obtained in the form of the pole expansion

$$\mathbf{Y} = \underline{\widehat{\mathbf{T}}}^{\mathrm{T}}(\underline{\mathbf{M}} + s\underline{\mathbf{N}})^{-1}\underline{\widehat{\mathbf{T}}} = \sum_{m=1}^{\underline{M}} \frac{\underline{\widehat{\mathbf{T}}}^{\mathrm{T}}\mathbf{y}_m\mathbf{y}_m^{\mathrm{T}}\underline{\widehat{\mathbf{T}}}}{\lambda_m + s}$$

where $\underline{M}$ ($\ll M$) is the order of the reduced-order model and $\lambda_m, \mathbf{y}_m$ are the eigenvalues and the eigenvectors obtained from the solution of the generalized eigenvalue problem $(\underline{\mathbf{M}} + \lambda\underline{\mathbf{N}})\mathbf{y} = 0$.

III. Example

Fig. 4 and Fig. 5 show two examples of application of the described algorithm. The results are reported in the form of scattering parameters, that are determined straightforwardly from the pole expansion of the admittance matrix. In both cases the basis functions were rectangular roof-tops. In the figures the results of our method are compared with the results obtained by a commercial code (EMSight™), which determines the scattering parameters, frequency by frequency, using the integral equation method in the spectral domain. The reported omputing times refer to a standard PC with a 1300 MHz AMD Athlon processor.

In the case of the directional coupler of Fig. 4 we used $K = 516$ basis functions, $f_{max} = 60$ GHz and $\zeta = 2.5$. We

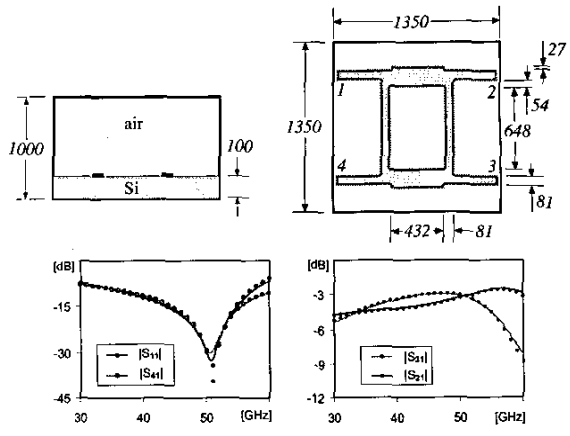

Fig. 4. Branch-line directional coupler on a Si substrate (dimensions in μm). $Z = 0.05\Omega$, $\epsilon_1 = 11.76$, $\sigma_1 = 1/30$ S/m. Solid line: this method; dots: EMSight results.

found only one complex pole-pair inside the circle C ($M' = 1$), whereas the number of real poles was $M'' = 3600$. The final order of the system was $\underline{M} = 42$. The computing time was about 40 s, whereas EMSight required about 70 s to calculate the 31 frequency samples that are reported in the figure. Taking into account the fact that our algorithm yields the *whole* frequency response from 0 to f_{max}, the time saving can be considered very significant.

In the case of the coupled-line band-pass filter of Fig. 5 we used $K = 241$ basis functions, $f_{max} = 70$ GHz and $\zeta = 2.5$. The number of complex poles was $M' = 10$ and the number of real ones was $M'' = 1500$. The final order of the system was $\underline{M} = 68$. The computing time was about 10 s, which, in this case, was about one order of magnitude shorter than the time required by EMSight™. The same structure was modelled with the procedure described in [11]: the new procedure gave rise to a time saving of about 50% with respect to the old one.

It is finally worthy noting that the above computing times include the evaluation of the coefficients of the pole expansions (10), which affects the total CPU time by about 30%. When performing repeated analysis of circuits on the same stratification and included in the same shielding box, *e.g.*, for optimization procedures, the evaluation of these coefficients must be performed only once, thus further increasing the efficiency of the proposed method.

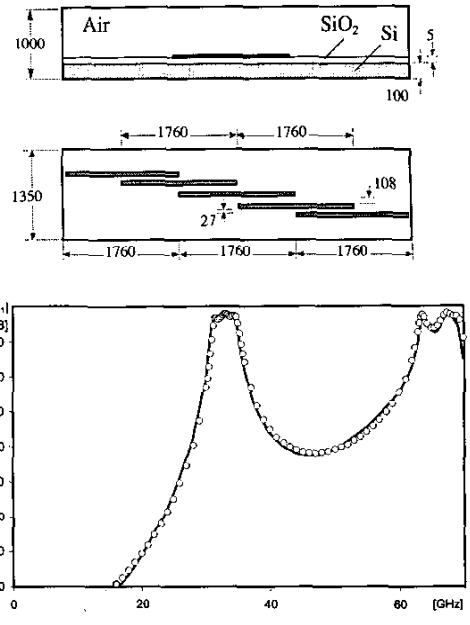

Fig. 5. Coupled-line 30 GHz bandpass filter on a Si/SiO_2 substrate (dimensions in μm, width of all strips = 81 μm). $Z = 0.05\Omega$, $\epsilon_1 = 11.76$, $\sigma_1 = 1/30$ S/m, $\epsilon_2 = 3.9$, $\sigma_2 = 0$. Solid line: this method; dots: EMSight results.

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